[Question 1]

*(1.1) Determine whether the system*

*is degenerate. In the degenerate case, decide whether it has no solution or in definitely many solutions. If it has no solution, explain why, else find the general form of the solutions.*

[1] Rewrite the given system of differential equations

[2] Combine the Equations

We have:

*&*

[3] substitute x into [2]

*Therefore, equation (1) is consistent with the substitution.*

[4] substitute x into [2]

*Therefore, equation (1) is* ***inconsistent*** *with the substitution.*

*Therefore, the system is* ***inconsistent &*** *no values of 𝑥(𝑡) and y(t) that can simultaneously satisfy both differential equations.*

*(1.2) Solve the system*

*by using the elimination method (operator method).*

*Hint. Eliminate y first.*

[1] Rewrite the given system of differential equations

[2] Eliminate y

[3] Solve for

[4] Substitute into [2]

[5] Solve for y

[6] Substitute into [1]

[7] Solve for x

[Question 2]

*Show that the system:*

*is equivalent to both the following triangular systems:*

*and*

[1] Rewrite the given system of differential equations

[2] Solve for y

[3] Solve for x

[4] substitute y into [2]

Which matches in the first triangular system.

[Question 3]

*Solve the system:*

*by using the elimination method (operator method).*

[1] Rewrite the given system of differential equations

[2] Solve for

[3] Solve for

[4] substitute into [1]

[5] solve for

Find complementary function (homogeneous solution)

*(characteristic polynomial)*

*or or*

complimentary function using roots:

where c1, c2 and c3 are arbitrary constants

[6] solve for

Find particular function (particular solution)

Assume

So,

Then,

Thus, particular solution is

Therefore, the general solution for is:

[6] solve for

Substitute [1] into

Thus,

ands

[Question 4]

*(4.1) Use the eigenvalue-eigenvectors to solve the initial value problem*

*,*

[1] find *eigenvalues*

*or*

[2] find *eigenvectors for*

*Thus,*

[3] find *eigenvectors for*

*Thus,*

[4] general solution

function using roots:

*or*

Thus,

where c1 and c2 are arbitrary constants

Substitute eigenvalues and eigenvectors:

[5] use Initial conditions

*Thus, we have*

And also,

[6] find &

Rewrite the given system of differential equations

Add [1] + [2]

Substitute into [1]

[7] general solution

Substitute & into general solution

Thus,

*(4.2) Use the eigenvalue-eigenvectors to solve the initial value problem*

*,*

[1] find *eigenvalues*

*upper triangular matrix, thus*

*or*

[2] find *eigenvectors for*

*Thus,*

*Therefore,*

[3] find *eigenvectors for*

*Thus,*

*Therefore,*

[4] general solution

function using roots:

*or*

Thus,

where c1, c2 and c3 are arbitrary constants

Substitute eigenvalues and eigenvectors:

[5] use Initial conditions to find , &

*Thus, we have*

And also,

[7] general solution

Rewrite the given system of differential equations

Substitute , & into general solution

[Question 5]

[Question 6]

Solve the system:

*,*

*Why is there a unique solution to the above system?*

[1] find *eigenvalues*

*thus*

*or*

*or*

*or*

*or*

[3] find *eigenvectors*

[Question 7]